arXiv:1304.2194v1 [hep-ph] 8 Apr 2013

Pion couplings of the $\Delta(1232)$

Alfons J. Buchmann
*Institute for Theoretical Physics, University of Tübingen, D-72076 Tübingen, Germany**

Steven A. Moszkowski
Department of Physics and Astronomy, University of California, Los Angeles, CA 90095-1547, USA†

We calculate the strong couplings of pions to the $\Delta(1232)$ resonance using a QCD parameterization method that includes in addition to the usual one-quark also two-quark and previously uncalculated three-quark operators. We find that three-quark operators are necessary to obtain results consistent with the data and other QCD based baryon structure models. Our results are also in quantitative agreement with a model employing large D state admixtures to the N and Δ wave functions indicating that the πN and $\pi\Delta$ couplings are sensitive to the spatial shape of these baryons.

PACS numbers: 11.30.Ly, 12.38.Lg, 13.75Gx, 14.20.Gk

I. INTRODUCTION

Since its discovery [1] in pion (π)-nucleon (N) scattering the lowest excited state of the nucleon with spin $S = 3/2$ and isospin $T = 3/2$, called $\Delta(1232)$, has been important both for an understanding of nucleon ground state structure [2] and the nucleon-nucleon interaction [3]. In sufficiently energetic NN collisions one or both nucleons may be excited to the $\Delta(1232)$, a process that makes a significant contribution to many nuclear phenomena [4], for example, electromagnetic properties of light nuclei [5], three-nucleon forces [6], and the binding energy of nuclear matter [7]. In addition, Δ degrees of freedom are needed to explain the empirical cross sections for the $\pi N \to \pi N$ [8], $\pi N \to \pi\pi N$ [9, 10], and $NN \to NN\pi\pi$ [11] reactions including a novel resonance structure in the $p\,n \to d\pi^0\pi^0$ channel [12].

However, a quantitative assessment of the role of Δ degrees of freedom in nuclear physics has remained difficult due to the lack of detailed knowledge of even basic Δ properties. For example, the widely used additive quark model, which is based on the assumption that observables can be calculated using a sum of one-quark operators, underpredicts the experimental $N \to \Delta$ transition magnetic moment by about 30% [13] but slightly overpredicts the Δ^+ magnetic moment [14]. Another example involves the strong coupling constants $f_{\pi N\Delta}$ and $f_{\pi\Delta\Delta}$. The former determines the decay rate $\Delta \to N+\pi$, while the latter fixes the $N\Delta$ interaction strength in nuclei (see Fig 1). Here again, the additive quark model underpredicts the empirical $N \to \Delta$ transition coupling $f_{\pi N\Delta}$ by 20%, whereas it appears to overpredict the double Δ coupling $f_{\pi\Delta\Delta}$. A resolution of these discrepancies is necessary for a quantitative description of Δ degrees of freedom in nuclei [15–17].

Previously, the strong $\pi\Delta$ couplings were calculated with a QCD parameterization method, in which in addition to one-quark operators (additive quark model), two-quark operators were taken into account [18]. It was shown that two-quark contributions amount to a 20% increase of $f_{\pi N\Delta}$ with respect to the additive quark model results. Furthermore, the following relation between the πN, $\pi N\Delta$ and $\pi\Delta\Delta$ couplings was derived

$$f_{\pi^0 pp} - \frac{1}{4}\, f_{\pi^0\Delta^+\Delta^+} = \frac{\sqrt{2}}{3}\, f_{\pi^0 p\Delta^+}. \qquad (1)$$

This relation connects the elusive $f_{\pi^0\Delta^+\Delta^+}$ to the better known $f_{\pi^0 p\Delta^+}$ and $f_{\pi^0 pp}$ couplings. Throughout this paper we use the normalization conventions of Ref. [19] for the one-quark contributions, i.e., $f_{\pi\Delta\Delta} = (4/5)\, f_{\pi NN}$ and $f_{\pi N\Delta} = (6\sqrt{2}/5)\, f_{\pi NN}$ [20]. These theoretical relations, which are based on the additive quark model (see sect. 3), satisfy Eq.(1). On the other hand, if the empirical relation $f_{\pi^0 p\Delta^+} = 2.1\, f_{\pi^0 pp}$ is used in Eq.(1), one obtains $f_{\pi^0\Delta^+\Delta^+} = 0.04\, f_{\pi^0 pp}$. However, from the viewpoint of QCD sum rules [21] and $1/N_c$ expansion [22] both $f_{\pi^0\Delta^+\Delta^+}$ and $f_{\pi^0 pp}$ should be of the same order of magnitude.

The main purpose of this paper is to investigate if the inclusion of three-quark operators can resolve the discrepancies between theory and experiment for the $\pi\Delta$ couplings. Another motivation for this study comes from the work of Abbas [23], who found that large D-wave admixtures in the $N(939)$ and $\Delta(1232)$ quark wave functions reduce the additive quark model result for $f_{\pi\Delta\Delta}$ by 20% while they increase $f_{\pi N\Delta}$ by about the same percentage. This indicates that the strong $\pi\Delta$ couplings are sensitive to the spatial shape of the quark distribution in the nucleon and its first excited state. Therefore, it is of interest to study whether the two- and three-quark terms have an analogous effect on the strong $\pi\Delta$ couplings and may thus be interpreted as describing degrees of freedom leading to nonspherical geometrical shapes of the N and Δ baryons.

*Electronic address: alfons.buchmann@uni-tuebingen.de
†Electronic address: stevemos@ucla.edu

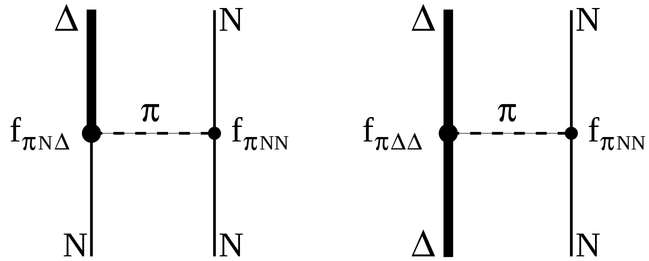

FIG. 1: Strong coupling of the pion to the nucleon (N) and Δ-isobar (Δ). The πNN, $\pi N\Delta$, and $\pi\Delta\Delta$ coupling constants are denoted as $f_{\pi NN}$, $f_{\pi N\Delta}$, and $f_{\pi\Delta\Delta}$. The corresponding interaction vertices are represented as black dots.

II. METHOD

As in our previous work we use a general parametrization (GP) method developed by Morpurgo and described in more detail in Refs. [24–26] to calculate the strong pion couplings. The most general expression for the corresponding operator $\mathcal{O}$ that is compatible with the space-time and inner QCD symmetries is a sum of one-, two-, and three-quark operators in spin-flavor space multiplied by *a priori* unknown constants (called A_1, A_2, and A_3 below), which parametrize the orbital and color space matrix elements. Empirically, a hierarchy in the importance of one-, two-, and three-quark operators is found. This fact can be understood in the $1/N_c$ expansion where two- and three-quark operators are usually suppressed by powers of $1/N_c$ and $1/N_c^2$ respectively compared to one-quark operators [27]. The two- and three-quark contributions are an effective description of gluons and quark-antiquark degrees of freedom that have been eliminated from the QCD wave function [24].

For the strong πN and $\pi\Delta$ couplings one-, two-, and three-quark axial vector operators are defined as

$$\begin{aligned} \mathcal{O}_1 &= A_1 \sum_i \tau_3^i \sigma_z^i, \\ \mathcal{O}_2 &= A_2 \sum_{i \neq j} \tau_3^i \sigma_z^j \\ \mathcal{O}_3 &= A_3 \sum_{i \neq j \neq k} \tau_3^i \, \sigma_z^i \, \boldsymbol{\sigma}^j \cdot \boldsymbol{\sigma}^k, \end{aligned} \tag{2}$$

and the total operator reads

$$\mathcal{O} = \mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3. \tag{3}$$

Here, $\boldsymbol{\sigma}^i$ and $\boldsymbol{\tau}^i$ are the spin and isospin operators of quark i.

In Ref. [18] we briefly discussed why the two-body operator $\mathcal{O}_2$ in Eq.(2) is unique. With respect to the three-quark operator $\mathcal{O}_3$ the reader may wonder why other three-quark operators, for example

$$\begin{aligned} \widetilde{\mathcal{O}}_3 &= \sum_{i \neq j \neq k} \tau_3^i \left[\boldsymbol{\sigma}^i \times \boldsymbol{\sigma}^j \times \boldsymbol{\sigma}^k \right]_z \\ \widehat{\mathcal{O}}_3 &= \sum_{i \neq j \neq k} \tau_3^i \, \sigma_z^j \, \boldsymbol{\sigma}^i \cdot \boldsymbol{\sigma}^k \end{aligned} \tag{4}$$

can be excluded from the list of permissible operators. It turns out that the operator $\widetilde{\mathcal{O}}_3$ is identical to zero when summed over quark indices. Furthermore, the operator $\widehat{\mathcal{O}}_3$ has for the spin-flavor symmetric N and Δ states considered here, the same matrix elements as the two-body operator $\mathcal{O}_2$ in Eq.(2) so that its effect is already included [28]. This is an example of an SU(6) operator reduction rule. More generally, SU(6) operator reduction rules [22] express the fact that seemingly different operators are not necessarily linearly independent on a given SU(6) representation. For example, $\widehat{\mathcal{O}}_3$ and $\mathcal{O}_2$ are linearly dependent when applied to the spin-flavor symmetric ground state **56**-plet.

The existence of unique one-, two-, and three-quark operators can also be understood from the following group theoretical argument. From the viewpoint of broken SU(6) spin-flavor symmetry both the N and Δ belong to the same **56** dimensional ground state multiplet. Therefore, an allowed symmetry breaking operator $\mathcal{O}$ must transform according to one of the irreducible representations found in the product

$$\bar{\mathbf{56}} \times \mathbf{56} = \mathbf{1} + \mathbf{35} + \mathbf{405} + \mathbf{2695}. \tag{5}$$

Here, the **1**, **35**, **405**, and **2695** dimensional representations, are respectively connected with zero- (a constant), one-, two-, and three-body operators. Because each representation on the right-hand side of Eq.(5) occurs only once, the operators in Eq.(2) are unique in the sense that for each $\mathcal{O}_i$ there is only one linearly independent operator structure. As a result, the operators in Eq.(2) provide a complete spin-flavor basis for the observables considered here.

baryon	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$
p	$\frac{5}{3}A_1$	$-\frac{2}{3}A_2$	$-\frac{26}{3}A_3$
$p \to \Delta^+$	$\frac{4\sqrt{2}}{3}A_1$	$-\frac{4\sqrt{2}}{3}A_2$	$\frac{8\sqrt{2}}{3}A_3$
Δ^+	A_1	$2A_2$	$2A_3$

TABLE I: Matrix elements $\mathcal{M}_i$ of the one-quark ($\mathcal{O}_1$), two-quark ($\mathcal{O}_2$), and three-quark ($\mathcal{O}_3$) axial vector operators in Eq.(2). The matrix element of the total operator $\mathcal{O}$ in Eq.(3) is $\mathcal{M}_q = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3$.

III. RESULTS

Evaluating the operators in Eq.(2) between SU(6) wave functions [29] for the N and Δ we get the results compiled in Table I.

To obtain from the total quark level matrix elements $\mathcal{M}_q$ in Table I the conventional pion-baryon couplings, one proceeds as follows [19]. Conventionally, the $N \to \Delta$ transition vertices depicted in Fig. 1 (left) are defined as matrix elements $\mathcal{M}_B$ of baryon level $N \to \Delta$ transition spin $\mathbf{S}$ and isospin $\mathbf{T}$ operators. The latter are normalized so that their matrix elements are equal to the corresponding spin and isospin Clebsch-Gordan coefficients

$$\begin{aligned}\mathcal{M}_B &= f_{\pi N\Delta}\,\langle \Delta, S'\,S'_z\,T'\,T'_z|\mathbf{S}_z\,\mathbf{T}_z|N, S\,S_z,\,T\,T_z\rangle\\ &= f_{\pi N\Delta}\,(1\,0\;S\,S_z|S'\,S'_z)\;(1\,0\;T\,T_z|T'\,T'_z). \qquad (6)\end{aligned}$$

Here, $S=T=1/2$ refers to the spin and isospin of the N, and $S'=T'=3/2$ to the spin and isospin of the Δ. For the $p\to\Delta^+$ transition with $T_z=T'_z=1/2$ and $S_z=S'_z=1/2$ Eq.(6) gives $\mathcal{M}_B = f_{\pi^0 p\Delta^+}(2/3)$. This baryon level matrix element must be equal to the corresponding total quark level matrix element, i.e., $\mathcal{M}_B = f_{\pi^0 p\Delta^+}(2/3) = \mathcal{M}_q$. Consequently, the $f_{\pi^0 p\Delta^+}$ coupling is obtained by multiplying the total quark level matrix element $\mathcal{M}_q$ in the second row of Table I by 3/2.

Analogously, $\Delta\Delta$ vertices depicted in Fig. 1 (right) are obtained from the diagonal matrix elements

$$\begin{aligned}\mathcal{M}_B &= f_{\pi\Delta\Delta}\,\langle \Delta, S'\,S'_z\,T'\,T'_z|\mathbf{S}_z\,\mathbf{T}_z|\Delta, S'\,S'_z\,T'\,T'_z\rangle\\ &= f_{\pi\Delta\Delta} S'_z\,T'_z. \qquad (7)\end{aligned}$$

Evaluating Eq.(7) for the Δ^+ with $T'_z=1/2$ and maximal spin projection $S'_z=3/2$ gives $\mathcal{M}_B = f_{\pi^0\Delta^+\Delta^+}(3/4)$. This baryon level matrix element must be equal to the corresponding total quark level matrix element, i.e., $\mathcal{M}_B = f_{\pi^0\Delta^+\Delta^+}(3/4) = \mathcal{M}_q$. Therefore, the entry for Δ^+ in Table I must be multiplied by 4/3.

We then get

$$\begin{aligned}f_{\pi^0 pp} &= \frac{5}{3}\,A_1 - \frac{2}{3}\,A_2 - \frac{26}{3}\,A_3,\\ f_{\pi^0 p\Delta^+} &= \frac{4\sqrt{2}}{3}\,(A_1 - A_2 + 2A_3)\left(\frac{3}{2}\right),\\ f_{\pi^0\Delta^+\Delta^+} &= (A_1 + 2\,A_2 + 2\,A_3)\left(\frac{4}{3}\right). \qquad (8)\end{aligned}$$

Solving Eq.(8) for the constants A_i leads to

$$\begin{aligned}A_1 &= \frac{1}{6}\,f_{\pi^0 pp} + \frac{\sqrt{2}}{9}\,f_{\pi^0 p\Delta^+} + \frac{5}{24}\,f_{\pi^0\Delta^+\Delta^+},\\ A_2 &= \frac{1}{4}\,f_{\pi^0\Delta^+\Delta^+} - \frac{\sqrt{2}}{12}\,f_{\pi^0 p\Delta^+},\\ A_3 &= -\frac{1}{12}\,f_{\pi^0 pp} + \frac{\sqrt{2}}{36}\,f_{\pi^0 p\Delta^+} + \frac{1}{48}\,f_{\pi^0\Delta^+\Delta^+}. \qquad (9)\end{aligned}$$

Next, we calculate numerical values for the strong Δ couplings, including successively first, second, and third order SU(6) symmetry breaking terms represented respectively by the operators $\mathcal{O}_1$, $\mathcal{O}_2$, and $\mathcal{O}_3$ in Eq.(2).

First, ignoring two- and three-quark terms, we get $A_1 = (3/5)\,f$, where we use the abbreviation $f := f_{\pi^0 pp}$. Thus, A_1 is fixed by the empirical value for the strong πNN coupling $f^2_{\pi^0 pp}/(4\pi) = 0.08$. In this first order SU(6) symmetry breaking approximation, we reproduce the well known additive quark model results for the $\pi\Delta$ couplings [19]

$$\begin{aligned}f_{\pi N\Delta} &= \frac{6\sqrt{2}}{5}\,f,\\ f_{\pi\Delta\Delta} &= \frac{4}{5}\,f. \qquad (10)\end{aligned}$$

Second, if we include two-quark but still neglect three-quark terms we need the empirical relation $f_{\pi N\Delta} = 2.1\,f$ to fix the additional constant A_2. In this case, SU(6) symmetry is broken up to second order. Eq.(8) with $A_3 = 0$ gives then $A_1 = 0.51\,f$ and $A_2 = -0.24\,f$ as in Ref. [18]. In this approximation we recover Eq.(1), which as shown in Table II entails an unrealistically small value for $f_{\pi^0\Delta^+\Delta^+}$.

Finally, the inclusion of three-quark terms takes third order SU(6) symmetry breaking into account. Using the QCD sum rule value $f_{\pi^0\Delta^+\Delta^+} = 0.666\,f$ [21], which is consistent with the data [9–11], allows us to fix the constant A_3 in Eq.(8). We then get from Eq.(9) the follwing values for the constants: $A_1 = 0.635\,f$, $A_2 = -0.081\,f$, and $A_3 = 0.013\,f$. By taking three-quark operators into account, we find that Eq.(1) is modified as follows

$$f_{\pi^0 pp} - \frac{1}{4}\,f_{\pi^0\Delta^+\Delta^+} = \frac{\sqrt{2}}{3}\,f_{\pi^0 p\Delta^+} - 12A_3. \qquad (11)$$

Consequently, $f_{\pi^0\Delta^+\Delta^+}$ can be of the same magnitude as $f_{\pi^0 pp}$ even when the empirical value for $f_{\pi^0 p\Delta^+}$ is used so that the discrepancy between theory and experiment can be resolved.

coupling	$\mathcal{M}_1$	$\mathcal{M}_1+\mathcal{M}_2$	$\mathcal{M}_1+\mathcal{M}_2+\mathcal{M}_3$	exp.
$f_{\pi^0 pp}$	1.00	1.00	1.00	1.00 [17]
$f_{\pi^0 p\Delta^+}$	1.70	2.10	2.10	2.10 [17]
$f_{\pi^0\Delta^+\Delta^+}$	0.80	0.04	0.67	0.06-1.72 [9]

TABLE II: Pion couplings of the $\Delta(1232)$ in terms of the pion-nucleon coupling $f = f_{\pi^0 pp}$ with the successive inclusion of one-quark ($\mathcal{M}_1$), two-quark ($\mathcal{M}_2$), and three-quark ($\mathcal{M}_3$) terms compared with experimental data. The experimental range for $f_{\pi^0\Delta^+\Delta^+}$ is from Table 9 in Ref. [9].

Table II lists the strong couplings $f_{\pi^0 pp}$, $f_{\pi^0 p\Delta^+}$, and $f_{\pi^0\Delta^+\Delta^+}$ in terms of f in the successive approximations discussed above. The numbers in the first column correspond to the additive quark model results in Eq.(10). The entries in the second column include the effect of the two-quark operator $\mathcal{O}_2$ of Ref. [18]. The latter changes the double Δ coupling from the additive quark model value $f_{\pi\Delta\Delta} = (4/5)f$ to $f_{\pi\Delta\Delta} = 0.04f$. However, such a small value for $f_{\pi\Delta\Delta}$ is inconsistent with other QCD based baryon structure models, which predict that $f_{\pi\Delta\Delta}$, $f_{\pi N\Delta}$, and $f_{\pi NN}$ are of the same order of magnitude. Finally, the third column represents a full calculation of one-, two-, and three-quark contributions. With three-quark terms included, the double Δ coupling changes

from $0.04f$ to $0.67f$, in qualitative agreement with QCD sum rule [21] and $1/N_c$ expansion [22] calculations. This provides evidence for the importance of three-quark operators in axial vector quantities, such as the pion-baryon couplings.

It is of interest to compare our results to those of Abbas [23], who found the following expressions for the $\pi N\Delta$ and $\pi\Delta\Delta$ couplings based on one-quark axial vector operators but with D state admixtures in the N and Δ wave functions

$$\begin{aligned} f_{\pi N\Delta} &= \frac{6\sqrt{2}}{5}\left[\frac{1-\frac{1}{2}P_D}{1-\frac{6}{5}P_D}\right]\frac{1}{\sqrt{1+P_D}}\,f, \\ f_{\pi\Delta\Delta} &= \frac{4}{5}\left[\frac{1-P_D}{1-\frac{6}{5}P_D}\,\frac{1}{1+P_D}\right]f, \end{aligned} \tag{12}$$

where P_D is the D-state probability in the nucleon wave function [30]. Note that the double Δ coupling in Eq.(12) has been adjusted to the normalization convention used here [20]. For $P_D = 0.34$ one finds $f_{\pi N\Delta} = 2.1f$ and $f_{\pi\Delta\Delta} = 0.67f$. Incidentally, the same value $P_D = 0.34$ also explains the empirically small quark contribution to nucleon spin [31].

As a result of the large D-state admixture in the nucleon and Δ wave functions, $f_{\pi N\Delta}$ is increased by 20%, and $f_{\pi\Delta\Delta}$ is decreased by the same percentage with respect to the additive quark model values in Eq.(10). This is consistent with the data and in quantitative agreement with the present results. Apparently, nonspherical N and Δ wave functions with large D-state probabilities have the same effect as the two- and three-quark contributions considered here.

This correspondence between large D wave admixtures and many-quark operators is not a coincidence. Via a unitary transformation it is always possible to eliminate many-body operators at the expense of more complicated wave functions without changing the observable matrix element [32]. Yet, the inclusion of many-quark operators leads overall to a more consistent and realistic description of nucleon structure for the following reasons. First, ab initio quark model calculations based on gluon and pion exchange potentials feature much smaller D state probabilities (< 0.01) [33] for the N and Δ wave functions. Second, many-quark operators are related to the quark-antiquark degrees of freedom, commonly refered to as meson cloud in physical baryons. Baryon deformation is more likely a result of these non-valence quark degrees of freedom than a consequence of massive valence quarks moving in elliptical orbits [34].

In any case, the ability of both models to describe the strong $\pi\Delta$ couplings is closely tied to nonspherical geometric shapes of the nucleon and Δ. Other observables, such as the $N \to \Delta$ quadrupole transition moment, baryon octupole moments, and the quark contribution to nucleon spin [34] point to the same conclusion concerning the nonsphericity of both N and Δ states.

IV. SUMMARY

In summary, we found that previously uncalculated three-quark operators make a significant contribution to the $\pi N\Delta$ and $\pi\Delta\Delta$ coupling constants. In particular, for the double Δ coupling, the three-quark term reduces the influence of the negative two-quark contribution so that the final result $f_{\pi^0\Delta^+\Delta^+} = 0.67\,f$ is about 20% smaller than the additive quark model value consistent with data and other QCD based baryon structure models.

Furthermore, our theory is in quantitative agreement with results obtained in a quark model with a large D-state admixtures. Both approaches increase $f_{\pi N\Delta}$ and simultaneously decrease $f_{\pi\Delta\Delta}$ by about 20% with respect to the additive quark model. This indicates that nonspherical N and Δ states are necessary for a quantitative understanding of the strong pion couplings independent of whether the spatial deformation is described as large D state components in the valence quark wave functions or (more realistically) as two- and three-quark operators representing a nonspherical sea of quark-antiquark pairs.

Having demonstrated the importance of two- and three-quark terms for a consistent description of the πN and $\pi\Delta$ couplings it will be interesting to investigate their effect on other axial vector quantities, e.g. the $p \to \Delta^+$ transition magnetic moment and the weak axial $N \to \Delta$ transition [35] for which large discrepancies between the additive quark model and experiment persist.

[1] H. L. Anderson, E. Fermi, A. A. Long, D. E. Nagle, Phys. Rev. **85**, 936 (1952).

[2] V. Pascalutsa, M. Vanderhaeghen, S. N. Yang, Phys. Rep. **437**, 125 (2007).

[3] R. B. Wiringa, R. A. Smith, T. L. Ainsworth, Phys. Rev. C **29**, 1207 (1984), R. Machleidt, Adv. Nuc. Phys. **19**, 189 (1989).

[4] Steven A. Moszkowski, in Proceedings of the 11th International Workshop on Condensed Matter Theories, July 27 - August 1, 1987, Oulu, Finland, ed. J. S. Arponen et al., Plenum Press, New York, 1988.

[5] H. Arenhövel and H. G. Miller Z. Phys. **266**, 13 (1974).

[6] E. Epelbaum, H. Krebs, U.-G. Meißner, Nucl. Phys. **A 806**, 65 (2008).

[7] L. S. Ferreira, Nucl. Phys. **A519**, 269c (1990).

[8] N. Fettes and U.-G. Meißner, Nucl.Phys. **A 679**, 629 (2001).

[9] R. A. Arndt et al., Phys. Rev. D **20**, 651 (1979).

[10] H. Kammano and M. Arima, Phys. Rev. C **69**, 025206 (2004).

[11] T. Skorodko et al., Phys. Lett. **B** 695, 115 (2011); M. Bashkanov et al., Phys. Rev. Lett. **102**, 052301 (2009);

L. Alvarez-Ruso, E. Oset, E. Hernández, Nucl. Phys. **A 633**, 519 (1998).
[12] P. Adlarson et al., Phys. Rev. Lett. **106**, 242302 (2011).
[13] R. H. Dalitz and D. G. Sutherland, Phys. Rev. **146**, 1180 (1966).
[14] M. Kotulla et al., Phys. Rev. Lett. **89**, 272001 (2002); I. C. Kloet, D. B. Leinweber, A. W. Thomas, Phys. Lett. B **563** 157 (2003); A. J. Buchmann, E. Hernández, A. Faessler, Phys. Rev. C **55**, 448 (1997); A. J. Buchmann, Proceedings of the NSTAR 2000 Conference, 16-19 February 2000, Newport News, ed. V. Burkert et al., World Scientific, Singapore, pg.59.
[15] A. M. Green, Rep. Prog. Phys. **39**, 1109 (1976).
[16] H. J. Weber and H. Arenhövel, Phys. Rep. **36**, 277 (1978).
[17] T. Ericson and W. Weise, Pions and Nuclei, Clarendon Press, Oxford, 1988.
[18] A. J. Buchmann, E. M. Henley, Phys. Lett. **B 484**, 255 (2000).
[19] G. E. Brown and W. Weise, Phys. Rep. **C 22**, 279 (1975). See Appendix B.
[20] Other authors, e.g. Refs. [15, 23], use the normalization convention $f_{\pi\Delta\Delta} = (1/5)\, f_{\pi NN}$. This is obtained if $f_{\pi^0\Delta^+\Delta^+}$ is evaluated for Δ^+ spin projection $S_z' = 1/2$, which gives $\langle \Delta^+, S_z'=1/2 \,|\, \mathcal{O}_1 \,|\, \Delta^+, S_z'=1/2\rangle = A_1/3$, instead of $\langle \Delta^+, S_z' = 3/2 \,|\, \mathcal{O}_1 \,|\, \Delta^+, S_z' = 3/2\rangle = A_1$ for maximal spin projection $S_z' = 3/2$ as in Table I. In our normalization scheme (see sect.3) the $\pi\Delta$ couplings are independent of the considered spin and isospin channel.
[21] Shin-Lin Zhu, Phys. Rev. C **63**, 018201 (2000). The $g_{\pi\Delta\Delta}$ and $f_{\pi\Delta\Delta}$ couplings are related as $f_{\pi^0\Delta^+\Delta^+} = f_{\pi^0\Delta^{++}\Delta^{++}} = (m_\pi/(2M_\Delta))\, g_{\pi^0\Delta^{++}\Delta^{++}}$.
[22] R. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D **49**, 4713 (1994); Phys. Rev. D **51**, 3697 (1995).
[23] A. Abbas, J. Phys. G **17**, L181 (1991).
[24] G. Morpurgo, Phys. Rev. D **40**, 2997 (1989); Phys. Rev. D **40**, 3111 (1989).
[25] G. Morpurgo, Phys. Rev. Lett. **68**, 139 (1992); Phys. Rev. D **46**, 4068 (1992); G. Dillon and G. Morpurgo, Phys. Rev. D **53**, 3754 (1996).
[26] G. Dillon and G. Morpurgo, Phys. Lett. **B 448**, 107 (1999); Phys. Lett. **B 459**, 321 (1999); Z. Phys. C **73**, 547 (1997).
[27] A.J. Buchmann and R.F. Lebed, Phys. Rev. D **62**, (2000) 096005; A.J. Buchmann, J.A. Hester, and R.F. Lebed, Phys. Rev. D **66**, (2002) 056002.
[28] The operator $\widehat{\mathcal{O}}_3$ in Eq.(4) has nonvanishing matrix elements only between the spin (mixed) symmetric parts $|\chi_S\rangle$ of the **56**-plet wave functions, for which one has, e.g., $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\, |\chi_S\rangle = +1\, |\chi_S\rangle$ so that $\widehat{\mathcal{O}}_3$ reduces to $\mathcal{O}_2$ in Eq.(2).
[29] D. B. Lichtenberg, Unitary Symmetry and Elementary Particles, Academic Press, New York, 1978; F. E. Close, An introduction to Quarks and Partons, Academic Press, London, 1979.
[30] The D state probability in the Δ wave function $P_D(\Delta)$ is related to the D state probability in the nucleon wave function P_D as $P_D(\Delta) = 2P_D/(1 + P_D)$ [23]. This has been used to eliminate $P_D(\Delta)$ from the original expressions for the $\pi\Delta$ couplings.
[31] A. Abbas, J. Phys. G **15**, L73 (1989).
[32] J. L. Friar, Phys. Lett. B **59**, 145 (1975).
[33] A. Buchmann, E. Hernández, and K. Yazaki, Phys. Lett. B **269**, 35 (1991), Nucl. Phys. **A 569**, 661 (1993).
[34] A. J. Buchmann and E. M. Henley, Phys. Rev. C **63**, 015202 (2000); Phys. Rev. D **65**, 073017 (2002); Eur. Phys. J. A **35**, 267 (2008); Phys. Rev. D **83**, 096011 (2011).
[35] Shi-Lin Zhu, M. J. Ramsey-Musolf, Phys. Rev. D **66**, 076008 (2002); D. Barquilla-Cano, A. J. Buchmann, and E. Hernández, Phys. Rev. C **75**, 065203 (2007).